Suggested Solutions to: Resit Exam, Spring 2017 Industrial Organization August 24, 2016

This version: August 23, 2017

Question 1: Downstream Cournot competition in a vertically related market

To the external examiner: The students had not seen this exact model before, but it is of course based on material that they have seen in the course.

Part (a)

We can solve for the (subgame perfect) equilibrium by using backward induction. Thus, firm D_i chooses q_i so as to maximize $\pi_i = (1 - w - Q)\, q_i$. The first-order condition can be written as[1]

$$\frac{\partial \pi_i}{\partial q_i} = 1 - w - Q - q_i = 0. \tag{1}$$

It is stated in the question that the retailers all choose the same quantity, so we can impose symmetry on (1) and thus obtain

$$q(w) = \frac{1-w}{n+1}. \tag{2}$$

At stage (i) of the game, the upstream firm U anticipates (2) and therefore maximizes the following profit:

$$\pi_U = wnq(w) = \frac{n(1-w)\,w}{n+1},$$

which has its optimum at $w^* = \frac{1}{2}$. Plugging this wholesale price back into (2) yields the following subgame perfect equilibrium value of q_i:

$$q^* = q(w^*) = \frac{1}{2(n+1)}. \tag{3}$$

The equilibrium value of the retail price p is obtained by using (3) in the demand function:

$$p^* = 1 - nq^* = 1 - \frac{n}{2(n+1)} = \frac{n+2}{2(n+1)}. \tag{4}$$

[1] It is also easy to check that the second-order condition is satisfied.

By inspection of (4), $\lim_{n\to\infty} p^* = \frac{1}{2} > 0$. That is, as n becomes very large the equilibrium price does *not* converge to the marginal production cost, zero, but is always strictly higher than that. Indeed, one can check that $p^* > \frac{1}{2}$ for all n.

Part (b)

We can again solve for the (subgame perfect) equilibrium by using backward induction. It is stated in the question that the downstream firms that are only retailers choose the same quantity; denote this common quantity by q_r. Whenever $w > 0$, the only-retailers have a higher cost than $\widehat{U}$ has. We should therefore consider the possibility that, at the equilibrium, those retailers will leave the market (so $q_r = 0$). From the first-order condition in (1) we have that q_r satisfies

$$q_r = \left\{ \frac{1 - w - q_1}{n}, 0 \right\}. \tag{5}$$

The stage (ii) profits of $\widehat{U}$ can be written as $\pi_{\widehat{U}} = (1-Q)\, q_1 + w \sum_{j=2}^{n} q_j$. The first-order condition becomes

$$\frac{\partial \pi_{\widehat{U}}}{\partial q_1} = 1 - Q - q_1 = 0 \Leftrightarrow 2q_1 + (n-1)\, q_r = 1. \tag{6}$$

A pair (q_1, q_r) is part of an equilibrium of the stage (ii) game if and only if it solves the equation system (5) and (6). First investigate if and when the equation system has a solution with $q_r = 0$. Plugging $q_r = 0$ into (6) yields $q_1 = \frac{1}{2}$. Given $q_1 = \frac{1}{2}$, (5) tells us that we indeed have $q_r = 0$ if and only if $w \geq \frac{1}{2}$. Next investigate if and when the equation system has a solution with $q_r > 0$. Given $q_r > 0$, (5) tells us that the equality $q_1 + nq_r = 1 - w$ must hold. Solving the equation system consisting of this equality and (6), we obtain

$$q_1 = \frac{1 + (n-1)\, w}{n+1} \quad \text{and} \quad q_r = \frac{1 - 2w}{n+1}.$$

The last expression for q_r tells us that we indeed have $q_r > 0$ if and only if $w < \frac{1}{2}$. Overall, we have found that the stage (ii) equilibrium is such that:

$$(\widetilde{q}_1, \widetilde{q}_r) = \begin{cases} \left(\frac{1+(n-1)w}{n+1}, \frac{1-2w}{n+1}\right) & \text{if } w < \frac{1}{2} \\ \left(\frac{1}{2}, 0\right) & \text{if } w \geq \frac{1}{2}. \end{cases} \quad (7)$$

At stage (i), $\widehat{U}$ chooses w in order to maximize its profit, anticipating (7). The reduced-form profit can be written as

$$\widetilde{\pi}_{\widehat{U}} = [1 - \widetilde{q}_1 - (n-1)\,\widetilde{q}_r]\,\widetilde{q}_1 + w\,(n-1)\,\widetilde{q}_r. \quad (8)$$

It follows from (7) that, for $w \geq \frac{1}{2}$, the profit is independent of w (this is simply because with such a high wholesale price the only-retailers leave the market and so the exact level of w does not matter). Thus suppose $w < \frac{1}{2}$ and consider the question how the profit is affected by an increase in w. When taking the derivative of (8), we can ignore the effect of w that goes through $\widetilde{q}_1$ (as this is zero, due to the envelope theorem). We thus have

$$\begin{aligned} \frac{\partial \widetilde{\pi}_{\widehat{U}}}{\partial w} &= -(n-1)\,\widetilde{q}_1 \frac{\partial \widetilde{q}_r}{\partial w} + (n-1)\left[\widetilde{q}_r + w\frac{\partial \widetilde{q}_r}{\partial w}\right] \\ &= (n-1)\left[\widetilde{q}_r - (\widetilde{q}_1 - w)\frac{\partial \widetilde{q}_r}{\partial w}\right] \\ &= (n-1)\left[\frac{1-2w}{n+1} - \left(\frac{1-2w}{n+1}\right)\left(\frac{-2}{n+1}\right)\right] \\ &= \left[1 - 2w + 2\left(\frac{1-2w}{n+1}\right)\right] > 0, \end{aligned}$$

where the third equality uses (7). That is, the merged upstream firm's profit is increasing in w for all $w < \frac{1}{2}$. Moreover, as we concluded before, for $w \geq \frac{1}{2}$ the profit is constant w.r.t. w. This means that the profit is maximized by setting w equal to one-half or larger (any such value is enough to drive the only-retail firms out of the market, which is the optimal thing to do).

This means that the overall equilibrium values of the quantities are given by the second row of (7): $q_1^{**} = \frac{1}{2}$ and $q_r^{**} = 0$. The associated equilibrium retail price equals $p^{**} = 1 - \frac{1}{2} = \frac{1}{2}$.

Part (c)

The results in parts (a) and (b) tell us that the equilibrium price is strictly lower in the model with the merger. In this sense, the merger is pro-competitive.

One effect that works in the direction of making the merger anti-competitive is that initially there is a fair amount of competition in the downstream market (there are n competing firms there, where n could potentially be very large). Moreover, the merger between D_1 and U creates an incentive for the newly created firm to force the other downstream firms out of the market (the merged firm has the instrument to do this, since it sets the wholesale price w, and it has the incentive to do it, since it is also active in the downstream market). This is indeed what happens: The merger creates a retail monopoly. All else equal, this should be bad for the consumers and for total surplus.

However, there is also another, pro-competitive, effect of the merger, namely that the *double marginalization problem* is avoided.

- The double marginalization problem: The actions taken by the non-integrated downstream firms influence also the upstream firm's profits. Moreover, internalizing those external effects (which the firms would do after integration) helps also the consumers, not only the upstream firm's profits. In particular, the integrated firm will have a stronger incentive to lower the price, since both the downstream and upstream profits are positively affected by that. Also, a lower price helps consumers and the consumer surplus.

A priori, it does not seem to be clear which effect is the strongest. However, our analysis in the (a) and (b) parts shows that, in this setting, the second effect is the strongest one and overall the merger leads to a lower retail price (we should also expect total surplus to be larger thanks to the merger).

Question 2: Price competition with and without capacity constraints

To the external examiner: The students had seen the two models below before. They were discussed in a lecture and they are covered by the syllabus.

Part (a)

There is a unique Nash equilibrium of the game: each firm charges a price that equals its marginal cost, $(p_1^*, p_2^*) = (c, c)$. The students should of course prove their claims (the calculations/arguments that are needed are standard—see textbook or lecture notes).

Part (b)

Claim that we are asked to prove: Both firms charging the price $p^* = 1 - \overline{q}_1 - \overline{q}_2$ is a Nash equilibrium.

Proof of the claim

- We need to show that a firm cannot, given that the rival charges p^*, increase its profit by either choosing a price $p < p^*$ or a price $p > p^*$.
- First note that if both firms charge the price p^*, each of them earns a positive profit:

$$\pi_1 = p^* \overline{q}_1 > 0 \qquad \text{and} \qquad \pi_2 = p^* \overline{q}_2 > 0.$$

- If charging a *lower* price ($p < p^*$), firm 1 would be able to sell more. But since it is already operating at its full capacity (i.e., the capacity constraint is binding) it cannot produce more, so a lower price would not increase its profit.
- Could the firm gain by *increasing* its price? If charging a higher price ($p > p^*$), firm 1's profit would equal:

$$\pi_1(p) = p(1 - p - \overline{q}_2)$$

  (here we use the assumption of efficient rationing—see also Figure 1).

  - Differentiate w.r.t. p:

$$\frac{\partial \pi_1(p)}{\partial p} = 1 - 2p - \overline{q}_2.$$

  - Evaluate at $p = p^*$ ($\overset{\text{def}}{=} 1 - \overline{q}_1 - \overline{q}_2$):

$$\frac{\partial \pi_1(p)}{\partial p}\Big|_{p=p^*} = 1 - 2\overbrace{(1 - \overline{q}_1 - \overline{q}_2)}^{=p^*} - \overline{q}_2$$
$$= 2\overline{q}_1 + \overline{q}_2 - 1 \leq 0.$$

  - That is: increasing p, starting at p^*, would not raise profits.

- We have thus shown that given that the rival firm charges p^*, a firm cannot increase its profit by increasing or decreasing its price from p^*. This means that $(p_1, p_2) = (p^*, p^*)$ is a Nash equilibrium, which we were asked to prove.

Part (c)

- (i) Kreps and Scheinkman studied a two-stage game where the firms, in the first stage, simultaneously choose capacities $\overline{q}_i$ (at some cost). Then at stage 2, knowing each other's capacity, the firms simultaneously choose prices p_i.
- (ii) The result that they could show can be summarized as follows:
  - Suppose the demand function is concave and the rationing rule is the efficient one.
  - Then the outcome (i.e., the equilibrium capacities/quantities and the equilibrium price) of the two-stage game is the same as that of the corresponding one-stage Cournot game.
- (iii) The result is a celebrated one and many economists interpret it as a justification for thinking of Cournot games as a reduced form representation of the two-stage game described above. This is appealing, because the story in the two-stage game sounds plausible and realistic (in particular, in that story there is someone who actually sets the prices, in contrast to the Cournot model). At the same time, the outcome is not as unrealistic as in the Bertrand model, where the equilibrium involves marginal cost pricing even when there are only two firms. So the outcome of the two-stage game combines the good and appealing features of the Bertrand and Cournot models, while avoiding the drawbacks with each of those models. However, there are some caveats:

- The result obtained by Kreps and Scheinkman, which can be referred to as "*Cournot outcome in the two-stage game*", is **weaker** than our result under a) where we obtained the "*exact Cournot reduced form*". With the latter result, we actually get exactly the Cournot profit functions, $\pi_i^{net} = [1 - (\overline{q}_1 + \overline{q}_2)]\,\overline{q}_i - c_0\overline{q}_i$ (where c_0 is the investment cost). This means that we in that case can also study a version of the Cournot model with, for example, sequential quantity choices.
- The Kreps-Scheinkman result is not very robust to changes in the assumptions. For example, it relies critically on the assumption of the efficient-rationing rule.
- In more general settings, the capacity choices in the full game may serve important roles that are not captured by a reduced form. For example, firms with **private information** may want to use the capacity choices as **informative signals** to its rivals.

• Further discussion of the implications of Kreps-Scheinkman's result:

- The predictions and welfare results of the traditional Cournot model can be provided with foundations in some extreme cases.
- The two-stage game illustrates a broad idea that firms may want to choose non-price actions that soften price competition.
- In many applications the exact Cournot profit functions are not essential. Instead the key thing is that the best-response functions are downward-sloping—i.e., that the firms' choice variables are **strategic substitutes**:

$$\frac{\partial^2 \pi_i}{\partial \overline{q}_i \partial \overline{q}_j} = \frac{\partial^2 \left(\left[P\left(\overline{q}_i + \overline{q}_j\right) - c_0\right]\overline{q}_i\right)}{\partial \overline{q}_i \partial \overline{q}_j} = P' + P''\overline{q}_i < 0.$$

This may very well hold even if the "*exact Cournot reduced form*" does not hold (Kreps-Scheinkman assumed $P'' \leq 0$).

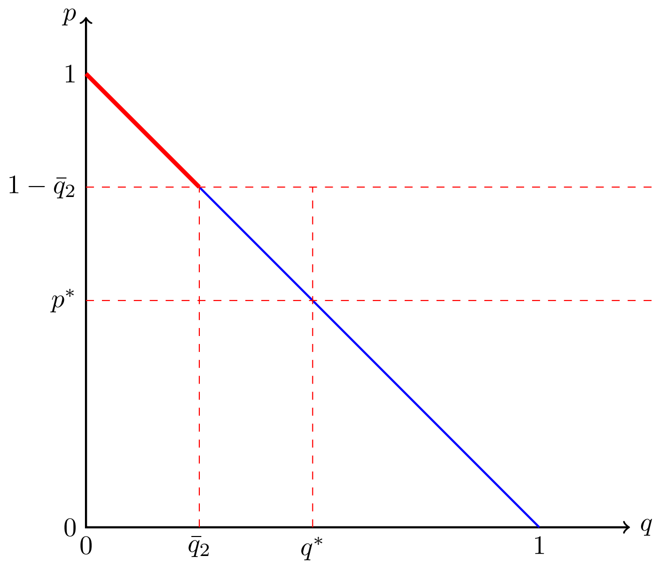

Figure 1: Efficient rationing